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FRC 100.009 v2.6: Operational Local Reciprocity Dynamics

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Abstract

> *FRC 100.009 v2.6 localizes the canonical relation dS + k* d ln C = 0 through an explicitly indexed operational realization. The representation k*_mu, entropy channel, coherence channel, units, and boundary are declared before evolution and are never refitted to evolving states. The local balance contains explicit environmental source and sink channels; the residual delta_R measures unresolved closure error or transient departure after represented exchange is included. It is not synonymous with openness or with a universal reciprocity current. The ARF throttle is a model-defined activation rule*

> *using the residual magnitude, so its pilot illustrates localized gating but cannot establish directional reciprocity.*

1. Introduction

The canonical FRC relation is

$$dS + k^* d \ln C = 0.$$

At a declared operating scale and boundary μ, the same law is instantiated as $dS_\mu + k^*_\mu d \ln C_\mu = 0$. Its use as a scale- and boundary-specific control relation is operational; its status as a universal open-system law remains conjectural. The toy model below does not settle reciprocity directionality. Its task is to localize storage, flux, represented exchange, and unresolved residual.

This paper develops a local field-level formulation of the **Adaptive Resonance Framework (ARF)**. We localize the global reciprocity law into a field continuity-style balance equation and treat the activation operator as derived from local reciprocity defect rather than inserted as an arbitrary constant floor.

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2. Core Mathematical Formulation

Let $x \in \Omega \subset \mathbb{R}^d$ denote position and $t \geq 0$ time.

2.1 Storage and Flux

We define the **local reciprocity storage density** as:

$$\Phi_R(x,t;\mu) = s_\mu(x,t) + k^*_\mu \ln\left(C_\mu(x,t) + \epsilon\right)$$

where the operating register, units, boundary, and k^*_μ representation are declared before evolution.

The **reciprocity flux** $J_R(x,t)$ is closed via a local gradient diffusion law:

$$J_R = -D_R \nabla \Phi_R$$

with diffusivity $D_R > 0$.

2.2 Local Reciprocity Defect

The open-system local **reciprocity defect** $\delta_R(x,t)$ measures the departure from local balance:

$$\delta_R(x,t) = \partial_t \Phi_R + \nabla \cdot J_R - \Sigma_R(x,t)$$

where the represented exchange is signed,

$$\Sigma_R = \Sigma_R^+ - \Sigma_R^-.$$

Σ_R^+ and Σ_R^- are explicit environmental source and sink channels. After those represented channels are included, δ_R measures unresolved closure error or transient departure. It is not synonymous with openness, entropy production, or the global reciprocity current.

To avoid circular dependency in numerical settings, the time derivative of storage is computed using a lagged backward finite-difference discretization:

$$\partial_t \Phi_R^n \approx \frac{\Phi_R^n - \Phi_R^{n-1}}{\Delta t}$$

2.3 ARF Gating Throttle

The Adaptive Resonance gating throttle $\mathcal{A}(x,t) \in [0,1)$ acts as a state-dependent activation or mobility operator:

$$\mathcal{A}(x,t) = \tanh\left(\kappa R(x,t) C(x,t) |\delta_R(x,t)|\right)$$

where $\kappa > 0$ represents defect sensitivity and $R(x,t) = 1$. Because the throttle uses $|\delta_R|$, it discards sign and cannot establish directional reciprocity.

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3. Minimal Numerical Toy Model

We illustrate this formulation by simulating the coupled base transport equations on a 1D grid ($N = 128$, $L = 1.0$):

$$\partial_t \rho = \mathcal{A}(x,t)\left[D_\rho \nabla^2 \rho - \chi \nabla \cdot \left(\frac{\rho}{1+\gamma\rho}\nabla C\right)\right] + \Xi_\rho$$

$$\partial_t C = \mathcal{A}(x,t)\left[D_C \nabla^2 C - \lambda_C (C - C_0) + \beta\rho\right] + \Xi_C$$

where γ is a chemotactic saturation parameter that prevents singular finite-time blow-ups.

3.1 Four Activation Regimes

The numerical simulations illustrate the self-starting properties of the chosen residual-gating rule:

1. **Regime A — Closed Silent Equilibrium:** Starting from a homogeneous state with no external source, the reciprocity defect remains zero ($\delta_R = 0$). The activation throttle remains completely dead ($\mathcal{A} = 0.0$), and the system remains frozen and silent.
2. **Regime B — Artificial Floor:** Injecting a small spatial wave perturbation and enforcing a 10% artificial activation floor drives evolution even in nominal equilibrium. This highlights the always-on character of the imposed floor.
3. **Regime C — Local Reciprocity Defect:** Starting from a highly localized initial density perturbation with no external source, the system exhibits localized activation around the defect. In the finite $t = 0.5$ run reported here, activation remains nonzero; eventual return to silence is a longer-time hypothesis, not a result of this pilot.
4. **Regime D - Represented source forcing:** Applying a weak localized source produces persistent activation and pattern formation. The existing run is source-only ($\Sigma_R^- = 0$); a signed source/sink balance remains a required extension.

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4. Observations & Shannon Entropy

Instead of a local provisional entropy-like density which can go negative, we define a normalized global Shannon entropy to track macrostate evolution:

$$p_i = \frac{\rho_i}{\sum_j \rho_j}, \quad S_\rho = -\sum_i p_i \ln(p_i + \epsilon)$$

During the source-forced run, $S_\rho(t)$ decreases as aggregation develops. This is a Shannon order diagnostic, not thermodynamic entropy production or dissipated heat.

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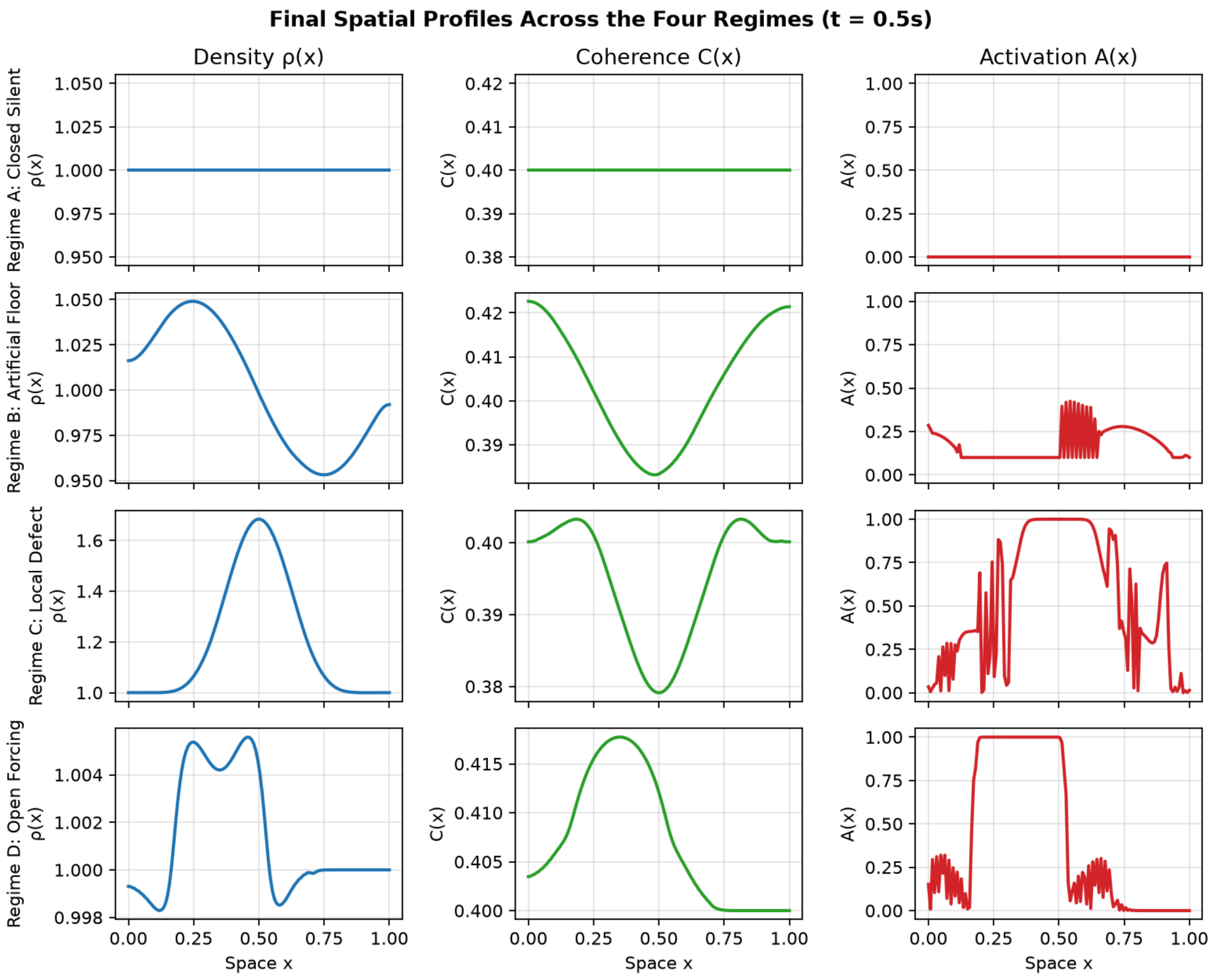
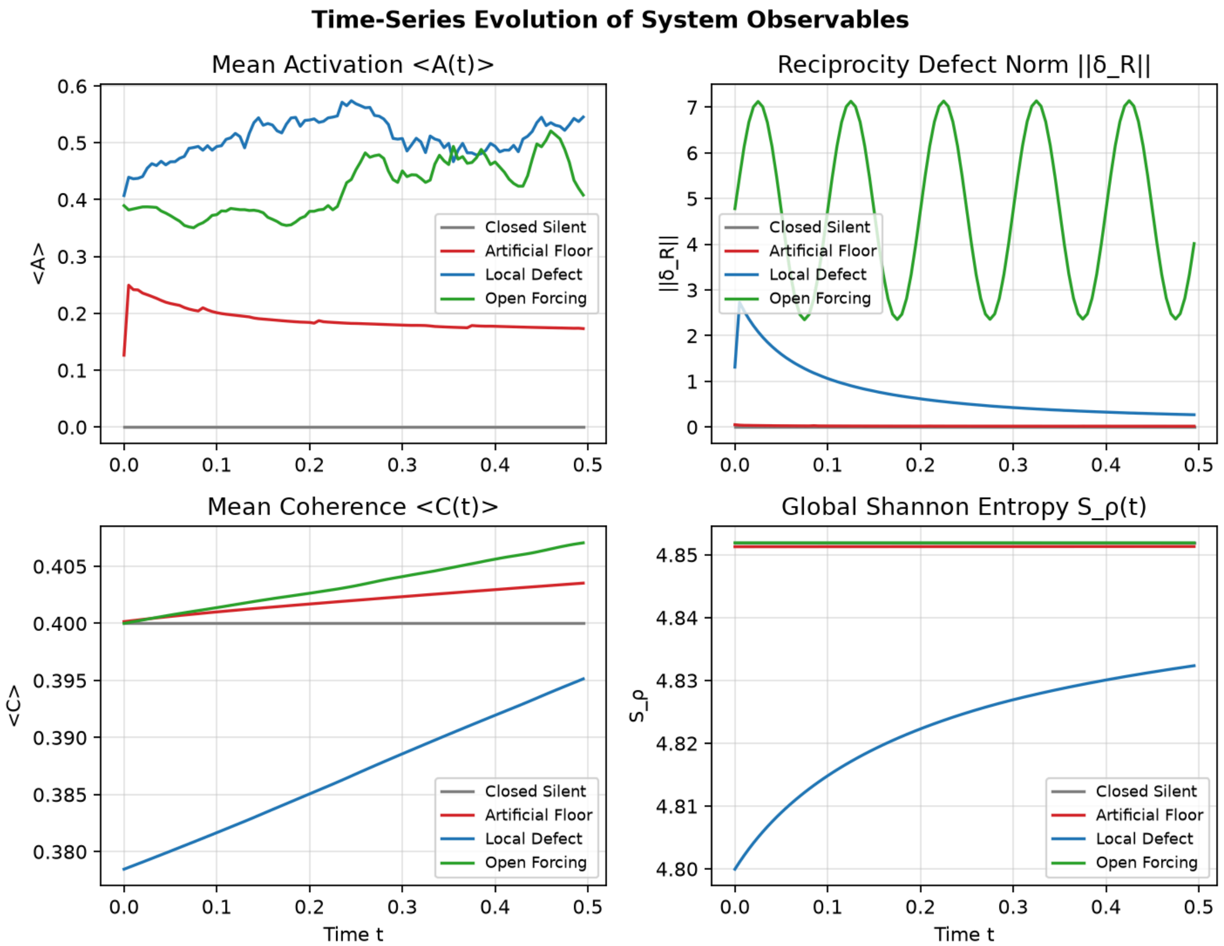
5. Reproducible Pilot

The fixed-parameter four-regime run produces:

Regime	Final mean activation	Final defect norm	Final mean coherence	Final Shannon entropy
Closed silent	0.000000	0.000000	0.400000	4.851902
Artificial floor	0.173185	0.019059	0.403531	4.851321
Local defect	0.545364	0.269426	0.395140	4.832339
Source-only forcing	0.408096	4.014826	0.407059	4.851899



Final density, coherence, and activation profiles for the four registered toy-model regimes.



Time-series comparison of activation, residual norm, mean coherence, and Shannon entropy.

These values illustrate the implemented throttle. They do not validate a thermodynamic closure or directional reciprocity. The source-only run uses $\Sigma_R^- = 0$; a signed source/sink experiment remains open.

The release includes `pilot/toy_model_100_009.py`, `results/toy_model_summary.csv`, `results/toy_model_timeseries.csv`, four PNG figures, and vector PDF copies. The script uses a fixed seed and recreates the complete result directory.

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6. Connections

- [[FRC-100-001]] — Original coherence definition and FRC operator
- [[FRC-100-007]] — observed, target, and optional latent Lambda objects
- [[FRC-566-001]] — Global entropy-coherence reciprocity law
- [[FRC-566-010]] — Universal Coherence Condition (UCC) flow formulation

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7. Version note

Version 2.6 restores the canonical/operational notation hierarchy without changing the pilot. It retains v2.5's removal of the settled-directional rewrite, separation of represented sources and sinks from unresolved residual, and narrow activation-rule interpretation.

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8. References

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4. Servat, H. (2025). *FRC 566.001: Entropy--Coherence Reciprocity*. Fractal Resonance Cognition Research Series.
5. Servat, H. (2026). *FRC 566.010: Universal Coherence Condition and Information-Geometric Field Flow*. Fractal Resonance Cognition Research Series.