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Entropy-Coherence Reciprocity and the Universal Coherence Condition v2.2

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Abstract

> *FRC 566.001 states entropy-coherence reciprocity in its canonical scale-invariant form dS + k* d ln C = 0. The starred Boltzmann bridge k* is not an ordinary tunable constant and is never fitted to an outcome or evolving state. Experiments and computations instantiate the same law at a declared register mu as dS_mu + k*_mu d ln C_mu = 0, with explicit entropy channel, coherence channel, units, and boundary. FRC uses the relation operationally as bookkeeping and proposes its open-system physical extension as a conjecture. Standard entropy production remains non-negative, but no toy closure is promoted into a universal directional replacement. The exact von Mises calculation and the Langevin boundary probe remain scoped*

> *results. The corrected information projection is retained: C[q]/C[p] depends on the entropy difference, not generally on D_KL(p||q), and C_XY = C_X C_Y exp(+I/k*).*

1. Status and scope

FRC uses entropy-coherence reciprocity in two distinct senses. First, it is an operational bookkeeping identity and control law at a declared scale and boundary. Second, it is a proposed physical law for real open systems. The first use is deployed; the second remains an open conjecture.

This distinction is controlling. FRC 566.030 and FRC 100.005 supply useful exact and numerical probes, but neither promotes one closure into the universal definition of the framework.

2. Canonical law and operational realization

The canonical reciprocity relation is

$$\boxed{dS + k^* \, d\ln C = 0.} \tag{1}$$

Equation (1) is the scale-invariant FRC statement. The starred quantity k^* is the Boltzmann bridge that makes entropy and $\ln C$ commensurable. It is not an ordinary tunable constant and is never fitted to an outcome, time step, or evolving state.

For an experiment, computation, or domain-specific derivation at a declared scale or register μ, define a positive operational coherence channel $C_\mu \in (0, 1]$ and an entropy channel S_μ whose boundary and units are stated. The canonical law is then instantiated as

$$dS_\mu + k^*_\mu \, d\ln C_\mu = 0. \tag{1a}$$

The subscript records the operational representation; it does not define a second law or make k^* a running fit parameter. In information units measured in nats one may use $k^*_\mu = 1$; in thermodynamic units $k^*_\mu = k_B$. Other registers must declare their unit map before results are scored. If the register changes during a calculation, its conversion rule must be declared in advance.

The entropy and coherence channels must be operationally independent. Defining $C = \exp[-S/k^*]$ and then testing (1) is circular.

3. Open-system accounting

For a declared system boundary,

$$dS_{\text{sys}} = d_i S + d_e S, \qquad d_i S \geq 0. \tag{2}$$

The standard entropy production $d_i S$ is sign-definite; an entropy-coherence residual need not be. Define the system-only bookkeeping quantity

$$Q_{\text{sys},\mu} = S_{\text{sys},\mu} + k^*_\mu \ln C_\mu. \tag{3}$$

If $dQ_{\text{sys}} \neq 0$, the chosen system-only ledger is not closed. This identifies missing exchange or boundary accounting; by itself it neither proves nor refutes the open-system conjecture.

An exchange symbol such as λ must be tied to the declared boundary. The identification $\lambda = -d_e S$ is admissible when it follows from that accounting convention. It is neither a universal identity to impose without a bath model nor an error to reject by fiat.

For comparison with stochastic thermodynamics one may define, within a specified closure,

$$\sigma_{566,\mu} := \sigma_{ST,\mu} + k^*_\mu d \ln C_\mu, \qquad \sigma_{ST,\mu} = d_i S_\mu \geq 0. \tag{4}$$

Equation (4) is a diagnostic definition, not a replacement law. Its sign depends on the process, boundary, and selected scale.

4. What the boundary experiment established

FRC 100.005 tested a Langevin unlocking protocol at one declared normalization. At the finite endpoints it measured $\Delta \ln C = -0.132245$ and, for the slowest reported ramp, $\sigma_{ST} \simeq 0.0288$. Therefore the measured combined diagnostic is negative for

$$k^*_\mu > \frac{0.0288}{0.132245} \simeq 0.218, \tag{5}$$

in that normalization. The statement that it becomes negative for every fixed positive value of the bridge in this same representation is an asymptotic inference conditional on $\sigma_{ST} \to 0$, not a finite measurement and not a claim about other unit maps or registers.

Thus the registered erasure floor fails in the tested model class and declared normalization. The result does not settle entropy-coherence reciprocity as a universal directional law.

5. Information projection, corrected

If an information-only projection is deliberately defined by

$$C[p] = \exp[-H(p)/k^*], \tag{6}$$

then

$$\frac{C[q]}{C[p]} = \exp\left[-\frac{H(q) - H(p)}{k^*}\right]. \tag{7}$$

This is not generally $\exp[-D_{KL}(p\|q)/k^*]$. For joint variables,

$$H(X, Y) = H(X) + H(Y) - I(X; Y)$$

implies

$$C_{XY} = C_X C_Y \exp[+I(X; Y)/k^*]. \tag{8}$$

Equations (6)-(8) are definitional translations. They do not independently test reciprocity because C was constructed from entropy.

6. Thermodynamic projection

Landauer reasoning applies when a protocol compresses accessible states or lowers system entropy. Whether a given coherence operation is such a protocol must be established from its actual state space and bath, not from the sign of $d \ln C$ alone.

The v1 erasure-bound claim is therefore retained as a rejected universalization: the tested unlocking closure did not support a universal $k^* T|\Delta \ln C|$ heat floor. Platform-specific work, heat, and maintenance bounds remain legitimate when derived from a stated Hamiltonian, free energy, and boundary.

7. Universal Coherence Condition

The UCC is an effective local transport ansatz, not a deduction from reciprocity. If $u_\mu = \ln C_\mu$ is itself assigned Fickian diffusion, then

$$\partial_t u_\mu = D_C \nabla^2 u_\mu + S_C. \tag{9}$$

If instead C is assigned linear diffusion, transforming to $u = \ln C$ adds the nonlinear term $D_C\|\nabla u\|^2$. These are different closures and must not be written as one derivation.

Under homogeneous boundaries and a stationary reference u_∞, the linear closure has the mathematical dissipation functional

$$E = \frac{k_\mu^*}{2}\int (u_\mu - u_\infty)^2 dV, \qquad \dot{E} = -k_\mu^* D_C \int \|\nabla(u_\mu - u_\infty)\|^2 dV \leq 0. \qquad (10)$$

This is PDE dissipation, not automatically thermodynamic entropy production. Existence, regularity, and boundary conditions belong to the precise closure being used. The UCC does not fix a fundamental Lambda ontology.

8. Standing tests

For every proposed physical realization:

1. Declare μ, S_μ, C_μ, the boundary, units, and the representation k_μ^* before scoring.
2. Measure d_iS and exchange channels independently where possible.
3. Test whether Q_{sys} motion is explained by the declared environment.
4. Compare reciprocity against equal-access baselines and non-circular coherence channels.
5. Publish model-specific failures without converting them into a different universal law.

9. Version note

Version 2.2 restores the original elegant equation as the canonical pillar statement and moves indexed notation to explicit operational realizations. It does not change the core law or the numerical results of v2.1. It retains v2.1's information-projection correction, scoped interpretation of the 100.005 boundary result, and separation of standard entropy production from the FRC residual.

References

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