UCC PDE: Existence, Uniqueness, and Dissipation

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Abstract

> *Establishes well-posedness for the Universal Coherence Condition (UCC) evolution of ln C — existence and uniqueness under standard boundary conditions via classical parabolic theory — and proves a dissipation inequality sigma(t) >= 0 for the coherence-gradient production term. Fractional variants are briefly discussed. Reproducible 1-D and 2-D numerical demos illustrate convergence to stationary states.*

1. Role in the 566 line

The 566 thermodynamics series develops entropy–coherence reciprocity from the founding law ([[FRC-566-001]]) into a partial-differential realization. This paper is the well-posedness step: it takes the Universal Coherence Condition (UCC) seriously as an evolution equation and asks whether that equation is mathematically sound — do solutions exist, are they unique, and does the coherence dynamics dissipate in the right direction?

[[FRC-566-020]] then linearizes this dynamics and derives modal relaxation rates; [[FRC-566-030]] completes the triplet by computing the entropy–coherence current across an exactly-solvable synchronization transition.

2. The equation

The UCC evolution for the log-coherence field is

$$\partial_t \ln C = D_C\, \Delta \ln C + S_C ,$$

posed on standard spatial domains with Neumann or Dirichlet boundary conditions, with $D_C > 0$ a coherence diffusivity and S_C a source term of stated regularity.

3. What the paper establishes

1. **Statement and assumptions.** The functional and spatial setting: domains, boundary conditions (Neumann/Dirichlet), and regularity assumptions on S_C.
2. **Existence and uniqueness.** Well-posedness by standard parabolic theory — energy estimates and the semigroup framework — with uniqueness from Gronwall-type arguments.
3. **Dissipation inequality and stationary solutions.** The coherence-gradient production term

$$\sigma(t) = k^{*} D_C \int_\Omega \|\nabla \ln C\|^2 \, dV \;\ge\; 0$$

is proven non-negative, and stationary solutions are characterized for simple choices of S_C.

1. **Fractional variants** are briefly discussed.

The mathematical machinery here is deliberately standard: the claim is not a new PDE technique, but that the UCC sits inside classical parabolic theory, so its existence, uniqueness, and dissipation structure are on solid ground. Note that UCC well-posedness for the full effective Lambda dynamics — with advection, relaxation, and positivity — remains listed as an open problem in the foundations paper [[FRC-826-829]]; this paper settles the core linear-diffusion-plus-source case.

4. Numerical demos (reproducible)

The release includes 1-D and 2-D demos (`code/566.010/pde_demo.py`) showing convergence to stationary states; figures regenerate from the script.

5. Record

- Zenodo record: [10.5281/zenodo.17526506](https://doi.org/10.5281/zenodo.17526506) (published 2025-11-04; includes `FRC_566_010_UCC_PDE_Wellposedness.pdf` and `release.zip` with the demo code).

Connections

- [[FRC-566-001]] — The founding entropy–coherence reciprocity law and UCC statement
- [[FRC-566-020]] — Spectral modes and relaxation: the linearized follow-up to this paper
- [[FRC-566-030]] — Reciprocity in action across a synchronization transition; third paper of the triplet
- [[FRC-100-009]] — Local reciprocity dynamics, which uses the UCC flow formulation
- [[FRC-826-829]] — Mathematical foundations; full effective-Lambda well-posedness remains an open problem there

References

1. Servat, H. (2025). *FRC 566.001: Entropy–Coherence Reciprocity*. Fractal Resonance Cognition Research Series.
2. Servat, H. (2025). *FRC 566.020: Spectral Modes and Relaxation in UCC*. DOI: 10.5281/zenodo.17526513.
3. Standard references on parabolic PDE theory, semigroups, and fractional Laplacians (as cited in the record PDF).