Spectral Modes and Relaxation in UCC

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Abstract

> *Linearizes the UCC evolution for u = ln C around stationary equilibria and derives modal relaxation rates tau_n = 1/(D_C lambda_n) from the Laplace spectrum of the domain. Includes 1-D and 2-D numerical demos with modal decay curves and relaxation-time tables, reproducible from the released code.*

1. Role in the 566 line

[[FRC-566-010]] established that the Universal Coherence Condition (UCC) evolution for $\ln C$ is well-posed and dissipative. This paper asks the natural next question: **how** does the coherence field relax toward its stationary states? The answer is spectral. [[FRC-566-030]] then completes the triplet by computing the entropy–coherence current across an exactly-solvable synchronization transition.

2. What the paper does

Writing $u = \ln C$, the UCC evolution

$$\partial_t u = D_C\, \Delta u + S_C$$

is linearized around stationary equilibria. Expanding the perturbation in eigenmodes of the Laplacian on the domain, with eigenvalues λ_n, each mode decays independently, and the modal relaxation rates follow directly from the Laplace spectrum:

$$\tau_n = \frac{1}{D_C\, \lambda_n} .$$

Slow, large-scale modes (small λ_n) relax slowly; fine-grained modes relax fast. The relaxation of coherence toward equilibrium is thus fully characterized by the domain geometry (through its Laplace spectrum) and the single diffusivity D_C.

3. Numerical demos (reproducible)

The release provides 1-D and 2-D numerical demos with modal decay curves and τ_n tables (`code/566.020/`, `artifacts/566.020/`), confirming the predicted mode-by-mode exponential decay against the analytic rates.

4. Why it matters downstream

The spectral picture is the quantitative content behind two later FRC claims:

- **Critical slowing** ([[FRC-826-829]], Proposition 3): as the leading relaxation eigenvalue approaches zero near a coherence transition, $\tau_{\rm relax}\sim1/|\mu_1|\to\infty$. The modal machinery here is where such rates are computed in practice.
- **Open reciprocity measurement** ([[FRC-566-030]]): relaxation timescales set the window in which the entropy–coherence exchange current can be resolved.

5. Record

- Zenodo record: [10.5281/zenodo.17526513](https://doi.org/10.5281/zenodo.17526513) (published 2025-11-04; includes `FRC_566_020_Spectral_Modes_and_Relaxation.pdf` and `release.zip` with demo code).

Connections

- [[FRC-566-001]] — The founding entropy–coherence reciprocity law
- [[FRC-566-010]] — UCC well-posedness and dissipation; the equation linearized here
- [[FRC-566-030]] — Reciprocity in action; third paper of the 566 triplet
- [[FRC-100-009]] — Local reciprocity dynamics using the UCC flow
- [[FRC-826-829]] — Foundations paper whose critical-slowing proposition rests on this modal picture

References

1. Servat, H. (2025). *FRC 566.010: UCC PDE — Existence, Uniqueness, and Dissipation*. DOI: 10.5281/zenodo.17526506.
2. Servat, H. (2026). *FRC 566.030: Reciprocity in Action*. DOI: 10.5281/zenodo.21264213.
3. Standard references on spectral theory of the Laplacian and linear parabolic relaxation (as cited in the record PDF).