Paper ID: FRC 566.030 | Version: v1.3

Reciprocity in Action v1.3: Exact System-Only Motion and a Scoped Boundary Test

Author: Hadi Servat

Affiliation: Independent Researcher

Contact: publish@fractalresonance.com

Website: fractalresonance.com

Abstract

> *FRC 566.030 applies the canonical reciprocity law dS + k* d ln C = 0 through one explicit information-unit realization. With the predeclared representation k*_{mu_nat}=1, it computes Q=S+k*_{mu_nat} ln C exactly on the von Mises/Kuramoto family. Q is non-constant and reaches its unique stationary point at kappa r=1, kappa=1.608279 and C=0.621782. The exact identity is dS/d ln C=-kappa r, hence dQ/d ln C=k*_{mu_nat}-kappa r. This stationary point is dQ=0; it is not sigma_566=0 unless an explicit environment model supplies that additional equality. The family contains no bath and therefore measures neither irreversible production nor entropy export. A companion Langevin closure tests those quantities in one model*

> *class and finds no universal erasure floor in its declared normalization.*

1. Question and status

This paper asks what the system-only quantity

$$Q_{\mathrm{sys},\mu} = S_{\mathrm{sys},\mu} + k_\mu^* \ln C_\mu \tag{1}$$

does across an exactly solvable synchronization family. It does not contain an environment model. Consequently it can compute Q_{sys} exactly, but it cannot decompose its motion into irreversible production and entropy exchange.

FRC uses reciprocity as bookkeeping at a declared scale and proposes its open-system physical extension as a conjecture. The calculation below is an exact exhibit for that program, not a standalone confirmation of the universal conjecture.

2. Operational normalization

The canonical FRC relation is $dS + k^* d \ln C = 0$. This calculation uses its operational realization in one declared information-unit register. The starred Boltzmann bridge is not fitted to the curve or recomputed from the evolving state.

Here entropy is measured in nats at one fixed information-unit scale, so

$$k_{\mu_{\mathrm{nat}}}^* = 1. \tag{2}$$

This value is declared before the sweep. It is not fitted to the curve.

3. Exactly solvable family

The von Mises density is

$$p(\theta;\kappa) = \frac{e^{\kappa\cos\theta}}{2\pi I_0(\kappa)}, \qquad \kappa \geq 0. \tag{3}$$

Its mean resultant length, used as the operational coherence channel, is

$$C(\kappa) = r(\kappa) = \frac{I_1(\kappa)}{I_0(\kappa)}. \tag{4}$$

Its differential entropy is

$$S(\kappa) = \ln\bigl(2\pi I_0(\kappa)\bigr) - \kappa r(\kappa). \tag{5}$$

Therefore

$$Q(\kappa) = S(\kappa) + k^*_{\mu_{\mathrm{nat}}} \ln r(\kappa). \tag{6}$$

4. Exact exchange-rate identity

Differentiating (4)-(5) gives

$$\frac{dS}{d\ln C} = -\kappa r. \tag{7}$$

Hence

$$\frac{dQ}{d\ln C} = k^*_{\mu_{\mathrm{nat}}} - \kappa r. \tag{8}$$

At the information-unit scale, $k^* = 1$, the unique stationary point satisfies

$$\kappa r = 1, \qquad \kappa = 1.608279, \qquad C = 0.621782. \tag{9}$$

Equation (9) means $dQ = 0$ along this statistical family. It does **not** imply $\sigma_{566} = 0$ without an environment model. If a separate closure defines

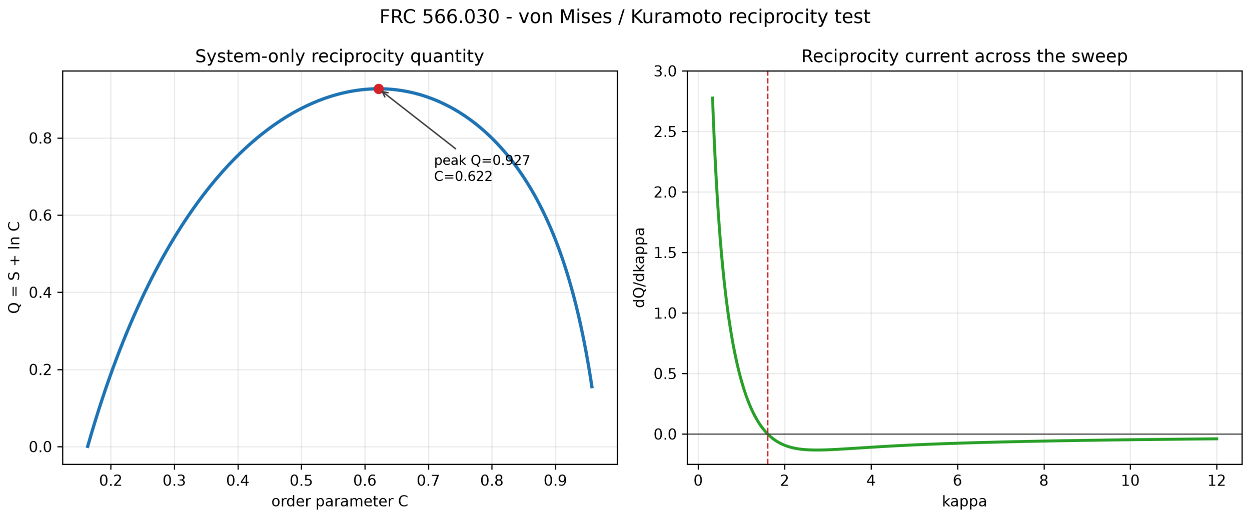
$$dQ = \sigma_{566} - dS_{\mathrm{env}}, \tag{10}$$

then $dQ = 0$ only yields $\sigma_{566} = dS_{\mathrm{env}}$.

5. Numerical values from the closed form

κ	C	S	$\ln C$	Q **for** $k^* = 1$

0.331	0.164	1.8109	-1.8105	0.0005
0.499	0.242	1.7783	-1.4178	0.361
1.003	0.447	1.6264	-0.8043	0.822
1.608	0.622	1.4049	-0.4774	**0.928**
1.998	0.697	1.2671	-0.3604	0.907
2.999	0.810	0.9936	-0.2109	0.783
3.999	0.864	0.8089	-0.1468	0.662
8.003	0.935	0.4139	-0.0669	0.347
12.000	0.957	0.1988	-0.0436	0.155



System-only Q and its derivative on the von Mises family. Q is non-constant and has one stationary point at kappa r = 1 for the declared information-unit scale. The curve does not separate production from environment exchange.

The non-constancy of Q_{sys} shows that the selected system-only ledger is not closed. It motivates an explicit boundary model. It is not, by itself, proof that one particular production/export decomposition is correct.

6. Why a constant-through-origin fit misleads

Forcing $S = -k \ln C$ on the system-only curve produces a strongly state-dependent apparent slope. This fit is not a measurement of k^* because it suppresses the offset, ignores the environment, and changes the accounting boundary.

The correct lesson is narrower: declare the operational representation before the sweep, compute Q_{sys}, and then measure the missing exchange channels independently. A normalization for another register cannot be inferred retrospectively from the same S-C curve being tested.

7. Scoped drive-and-bath result

FRC 100.005 later implemented one Langevin drive-and-bath closure. In that closure the finite slow-unlock run measured

$$\sigma_{ST} \simeq 0.0288, \qquad \Delta \ln C = -0.132245. \tag{11}$$

Thus its defined combined diagnostic $\sigma_{566} = \sigma_{ST} + k^* \Delta \ln C$ is negative at the finite endpoint only for $k^* > 0.218$ in that normalization. Negativity for every fixed positive k^* is an asymptotic inference conditional on $\sigma_{ST} \to 0$.

This rejects a universal erasure floor in the tested closure and normalization. It does not generalize the measured coefficient to another register and does not turn directionality into the replacement FRC law.

8. What remains open

A direct physical test must declare:

1. the operating register μ, units, and representation k^*_μ;
2. independent operational definitions of S_μ and C_μ;
3. a system boundary and measured dS_{env};
4. a production estimator independent of the reciprocity residual;
5. controls against algebraic identity and post-hoc scale fitting.

Only such a closure can determine whether an exchange residual such as λ equals $-d_e S$ under that boundary convention.

9. Version note

Version 1.3 restores the elegant canonical law in the pillar framing and keeps indexed notation only for this paper's explicit information-unit realization. It does not change any closed-form value or v1.2's scoped stationary-point interpretation.

Appendix - Reproducibility

The accompanying script `frc566030_reciprocity.py` evaluates (3)-(8), regenerates the table and figure, and locates (9). No fit or stochastic simulation enters this calculation.

References

1. H. Servat, *FRC 566.001 v2.2 - Entropy-Coherence Reciprocity and UCC*.
2. H. Servat, *FRC 100.005 v2.3 - Thermodynamics of Locking*.
3. Y. Kuramoto, *Chemical Oscillations, Waves, and Turbulence*, Springer (1984).
4. K. V. Mardia, P. E. Jupp, *Directional Statistics*, Wiley (1999).
5. I. Prigogine, *Introduction to Thermodynamics of Irreversible Processes*, Wiley (1967).