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Mathematical Foundations v1.5: Canonical Reciprocity and Status-Labeled Open Problems

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Abstract

> *FRC 826.829 states Fractal Resonance Coherence as a status-labeled mathematical research program. Version 1.5 restores the canonical scale-invariant relation dS + k* d ln C = 0 and reserves dS_mu + k*_mu d ln C_mu = 0 for declared operational realizations. The starred k* is the Boltzmann bridge, not an outcome-fitted constant or evolving state variable. The relation is operational bookkeeping; its physical universality for open systems remains a conjecture. Boundary-relative lambda=-d_eS is admissible when it follows from an explicit accounting convention, but lowercase lambda remains a diagnostic rather than a Lambda field. The paper distinguishes*

> *Lambda_obs, Lambda_eq, and optional Lambda_dyn from a separate fundamental-field conjecture. Four mathematical results are retained: a two-pole interior band, the conditional forced-cubic coefficient in a non-even coherence expansion, critical slowing, and the exact von Mises identity dS/d ln C=-kappa r. At the information-unit normalization k*_{mu_nat}=1, kappa r=1 is a stationary point of the system-only Q curve, not a physical zero-current claim without an environment model. Negative information-geometric and half-line results remain visible.*

1. Intent

This paper states the mathematical spine of Fractal Resonance Coherence. It is not a proof that the physical conjectures are true. It is a status-labeled map of the program.

Every claim is assigned one of four statuses:

Status	Meaning
Definition	a convention or mathematical object introduced by the framework
Proposition	a standard computation, symmetry argument, or directly checked identity
Conjecture	a physical claim not yet proven
Open problem	a precise task left for proof, disproof, or refinement

The discipline matters. A foundations paper should not hide conjectures inside theorem language. The aim here is that a mathematically trained reader can see exactly where to push.

2. Coherence And The Lambda Field

Let a system have phases $\phi_1, \ldots, \phi_N$ on the circle. A basic coherence parameter is the Kuramoto order-parameter magnitude

$$C = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\phi_j} \right|.$$

For the Lambda field we work on the positive codomain:

$$C \in (0, 1].$$

The instantaneous observed transform is

$$\Lambda_{\text{obs}} = \Lambda_0 \ln C_{\text{obs}}, \qquad \Lambda_0 > 0.$$

Thus $\Lambda_{\text{obs}} \leq 0$ when $C_{\text{obs}} \leq 1$. Full coherence corresponds to zero and the incoherent limit to $-\infty$.

Where a closure supplies a target, write $\Lambda_{\text{eq}} = \Lambda_0 \ln C_{\text{target}}$. An optional latent or coarse-grained state Λ_{dyn} may relax toward that target. These objects are not interchangeable, and none by itself proves a fundamental field.

The starred bridge $k^* > 0$ makes entropy and $\ln C$ commensurable. It is the Boltzmann bridge of the canonical relation, not a parameter refitted to each state or result. An operational register may label its representation as k^*_μ, but the label does not define a different law or a running state variable.

3. Reciprocity: Identity, Boundary Convention, and Conjecture

The canonical reciprocity relation is

$$dS + k^* d \ln C = 0.$$

For a declared scale, unit convention, and accounting boundary, the same relation is instantiated operationally as

$$dS_\mu + k^*_\mu d \ln C_\mu = 0.$$

The operational representation, entropy channel, coherence channel, and boundary must be declared before scoring. They may not be inferred from the outcome being tested.

For a modeled subsystem, the standard split is

$$dS_{\text{sys}} = d_i S + d_e S, \qquad d_i S \geq 0.$$

Define a boundary-relative residual

$$\lambda_{B,\mu} := d_i S_\mu + k^*_\mu d \ln C_\mu.$$

If the operational identity is applied to that same subsystem boundary, then

$$\lambda_{B,\mu} = -d_e S_\mu.$$

This is an admissible accounting convention. It is not universal absent a boundary model, but neither is it a closed-system error to reject categorically. If λ instead denotes total production, an expanded system-plus-environment residual, or another measured current, an explicit translation is required.

That the invariant form governs real open systems beyond a chosen bookkeeping convention remains an open physical conjecture. Model-specific tests constrain registered bounds within their model class; they do not alone settle the conjecture.

4. Effective Dynamics

At the effective level, an optional latent state is modeled by

$$\tau_\Lambda(\partial_t \Lambda_{\mathrm{dyn}} + u \cdot \nabla \Lambda_{\mathrm{dyn}}) = D_\Lambda \nabla^2 \Lambda_{\mathrm{dyn}} - \Lambda_{\mathrm{dyn}} + \Lambda_{\mathrm{eq}}[\rho, \psi, E] + \xi_\Lambda.$$

This places the effective dynamics in the reaction-diffusion and time-dependent Landau-Ginzburg family. It does not by itself prove that every physical system follows this form.

At a separate proposed fundamental level, one may write a scalar field Lagrangian

$$\mathcal{L}_\Lambda = \frac{1}{2}(\partial_\mu \Lambda)(\partial^\mu \Lambda) - V(\Lambda),$$

with

$$V(\Lambda) = \frac{1}{2} m_\Lambda^2 \Lambda^2 + \frac{\lambda_4}{4!}\Lambda^4.$$

The corresponding field equation has Klein-Gordon form with source terms. This branch is a conjecture, not a consequence of the operational transform or latent closure. It also has a known tension: the field theory ranges over real Λ, while the coherence definition requires $\Lambda \leq 0$. Section 11 records the quantified obstruction.

Order-parameter coordinate warning. In Λ coordinates, $-k^* \ln C = -k^* \Lambda/\Lambda_0$ is linear and contributes no cubic self-interaction. The forced cubic below concerns a different order parameter m for which $C = C(m)$ is non-even. A linear source in Λ coordinates and a cubic in m coordinates are not the same coefficient and must not be conflated.

5. Proposition 1: Two-Pole Interior Band

Let $F(C)$ be a bounded functional measuring integration that preserves differentiation, with $C \in (0, 1)$. In the class used by the FRC participation-ratio computations, both endpoints vanish:

$$F(0^+) = 0, \qquad F(1^-) = 0.$$

Therefore F attains an interior maximum on $(0, 1)$.

The established statement is the existence of an interior band, not a universal numerical coordinate for that band. In particular, no golden-ratio value is asserted by this proposition.

6. Proposition 2: Forced Cubic

Let a coherence functional near a reference point have the expansion

$$C(m) = 1 + c_1 m + c_2 m^2 + c_3 m^3 + O(m^4).$$

The reciprocity entropy term is

$$U(m) = -k^* \ln C(m).$$

The coefficient of m^3 is

$$a_3 = \frac{k^*}{3}(-c_1^3 + 3c_1 c_2 - 3c_3).$$

Unless the odd part of $\ln C(m)$ vanishes through third order, this reciprocity contribution contains a cubic. This conditional Taylor result is exact. It does not say that the total physical potential has an uncancelled cubic, that m is always the correct order parameter, or that the transition must be first order; those require the complete energetic sector and dynamics.

7. Proposition 3: Critical Slowing

Linearize the effective Lambda dynamics around a fixed point Λ_*. If μ_1 is the leading eigenvalue of the linearization, then near a coherence transition or bifurcation:

$$\mu_1 \to 0^-, \qquad \tau_{\text{relax}} \sim \frac{1}{|\mu_1|} \to \infty.$$

This is standard critical slowing. FRC's claim is not that this theorem is new. The FRC wager is that coherence transitions in selected physical, biological, or computational systems should show measurable slowing in the Lambda channel before collapse or state change.

8. Proposition 4: Von Mises Exchange-Rate Identity

The von Mises family is the maximum-entropy phase distribution with constrained mean resultant length:

$$p(\phi) = \frac{e^{\kappa \cos(\phi-\mu)}}{2\pi I_0(\kappa)}.$$

Its coherence is

$$r = C = \frac{I_1(\kappa)}{I_0(\kappa)}.$$

Its entropy is

$$S(\kappa) = \ln(2\pi I_0(\kappa)) - \kappa r.$$

Along this family,

$$\frac{dS}{d\ln C} = -\kappa r.$$

Therefore the system-only ledger differential at declared scale μ is

$$dQ_{\mu_{\rm nat}} = (k^*_{\mu_{\rm nat}} - \kappa r) d \ln C.$$

For the information-unit representation $k^*_{\mu_{\rm nat}} = 1$, $Q_{\mu_{\rm nat}}$ has one stationary point:

Quantity	Value
κ	1.608279
$C = r$	0.621782
κr	1.000000

This is the analytic form of the same peak used in FRC 566.030. It means $dQ_{\mu_{\rm nat}} = 0$ along this family. It does not identify a physical exchange residual or imply $\sigma_{566} = 0$ until a boundary and environment model are supplied.

9. Negative Result: Lambda Is Not A Canonical Coordinate In Two Named Senses

The von Mises family is a canonical statistical arena because it is an exponential family. Its natural coordinate is κ, its expectation coordinate is $r = C$, and its Fisher potential is $\ln I_0(\kappa)$ up to constants.

Two candidate claims fail on this family:

Candidate claim	Result
$\ln C$ is a dual-affine coordinate	negative; $r = C$ is the mixture-affine coordinate
$\ln C$ is the Fisher potential	negative; $\ln I_0(\kappa)$ is the potential whose Hessian gives the Fisher metric

This is an important loss. It removes a tempting route by which $\Lambda = \Lambda_0 \ln C$ could be called canonical. The surviving information-geometric question is narrower: whether the open production form

$$d_i S_\mu + k^*_\mu d \ln C_\mu$$

has a connection, curvature, or holonomy interpretation on a suitable statistical bundle.

10. Conjectures

The following are physical conjectures, not established results.

Conjecture	Current status
The canonical scale-invariant reciprocity form governs real open systems when its operational channels and boundary are declared	open physical conjecture; neither a state-by-state fit nor settled by one finite model
Collapse dynamics recovers the Born rule as an attractor	supported only in equilibrium-limit arguments; a rigorous H-theorem remains open
Specific constants sit at coherence-ease thresholds	high risk; only defensible where the threshold is independently defined

The paper deliberately labels these as conjectures. Their failure would not erase the definitions or the propositions.

11. Half-Line Obstruction

The coherence definition requires $\Lambda \leq 0$ when $C \leq 1$. A fluctuating real scalar field does not automatically respect that half-line.

The deterministic appendix computes the positive-tail probability for an unconstrained Gaussian fluctuation around a negative mean:

Mean Lambda	Fluctuation scale	Fraction with Lambda greater than 0
-0.99	0.22	0.000003
-0.51	0.42	0.112319
-0.57	0.69	0.204377
-0.34	0.57	0.275424

When the fluctuation scale becomes comparable to the distance from the boundary, the field spends order-ten-percent time outside the physical $\Lambda \leq 0$ region. This is not a cosmetic problem.

There are two possible paths:

1. derive the half-line as an effective coarse-grained constraint;
2. reparametrize the fundamental field so $\Lambda < 0$ intrinsically.

The second path costs the clean polynomial scalar theory. This is why the fundamental-to-effective problem remains open.

12. Open Problems

The program reduces to six precise open problems.

Open problem	Task
UCC well-posedness	prove existence, uniqueness, regularity, and positivity for the effective Lambda PDE
Born H-theorem	prove or disprove a Lyapunov functional whose equilibrium is the Born distribution
Connection or holonomy form	determine whether the open ledger is a connection or curvature object
Fundamental-to-effective coarse-graining	derive the effective UCC from a fundamental field while respecting $\Lambda \leq 0$
Forced-cubic universality class	classify systems where the forced cubic changes transition order
Two-pole bifurcation normal form	classify how the interior maximum appears and moves with scale and coupling

These are the points where the mathematical program can be advanced or killed.

13. Reproducibility Appendix

The release package includes:

`pilot/foundations_checks.py`

The script reproduces:

1. the symbolic forced-cubic coefficient formula;
2. the von Mises exchange-rate identity by finite-difference check;
3. the closure point $\kappa r = 1$;
4. the half-line obstruction fractions.

The canonical output is:

Check	Value
forced-cubic symbolic check	$a_3 = k^*(-c_1^3 + 3c_1c_2 - 3c_3)/3$
exchange identity median relative error	4.57e-7
exchange identity max relative error, trimmed	1.41e-6
closure κ for $k^* = 1$	1.608279
closure C for $k^* = 1$	0.621782

This appendix is not a substitute for the open problems. It is a guard against publishing purely verbal mathematics where a direct computation is available.

14. Conclusion

The mathematical content of FRC, stated carefully, is not a finished theory. It is a structured research program:

1. define coherence and the Lambda field;
2. keep the canonical law, operational instantiation, boundary diagnostic, and physical conjecture distinct;
3. place effective Lambda dynamics in known reaction-diffusion and Landau-Ginzburg families;
4. retain the established structural results;
5. record the negative results at the same volume as the confirmations;
6. leave the physical claims as conjectures until the open problems are solved or killed.

That is enough for a serious foundations paper. It gives the program a clean mathematical boundary and makes the next work visible.

15. Version note

Version 1.5 restores the elegant canonical relation and confines indexed notation to declared operational realizations. It retains v1.4's admissible boundary convention for $\lambda = -d_e S$, separation of observed, target, latent, and fundamental Lambda objects, removal of the orphaned numerical cubic example, and corrected von Mises stationary-point interpretation.

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